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DISCUSSION OF A STUDY OF END CONNECTIONS FOR STRUTS

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By Jack R. Benjamin, James W. Harland, F. P. Shearwood, S. K. Ghaswala, and Marshall Holt and J. W. Clark

STRUCTURAL DIVISION

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DISCUSSION

JACK R. BENJAMIN,[15] ASSOC. M. ASCE.—Excellent as this qualitative investigation is, it is unfortunate that a more quantitative study could not have been made. In all cases the end conditions are unknown. Only a general range is indicated. It is not reasonable to assume that the eccentricities were constant and the writer has found that this item cannot begin to be described by the terms, "round ends" or "flat ends." A much more extensive investigation is necessary before the data can be accepted as being valuable quantitatively.

Extension of strength data to practical structures is impossible on a rational basis. The tests given are only tests of the struts used, with end conditions peculiar to each in a particular testing machine. A much more detailed study must be made before engineers can begin to know what they are doing in the design of struts and their connections.

JAMES W. HARLAND,[16] JUN. ASCE.—Any investigation which points out possible new uses, or which makes additional information available concerning accepted practices, is to be commended, and the authors are certainly to be complimented for this paper.

It is regrettable that the photographs showing the specimens under load (Figs. 1, 2, and 3), did not show the details of the end connections more clearly.

The authors state that, although these tests were made on various aluminum alloys, the results are thought to be applicable to other materials as well. It is known that the principal structural material, steel, differs from most aluminum alloys in that it possesses much greater ductility. The property of ductility permits a material to yield and distribute load so that stress concentrations in a ductile material are not as severe as in a nonductile material. That is, a stress concentration detail would not be as serious in structural steel as it would be in an aluminum alloy specimen.

Another point at which steel and aluminum differ is in the value of Young's modulus (E). It is notable in Table 4, that 14 of the 26 specimens are listed as having failed wholly or partly as a result of lateral deflection. In a material such as steel, which has a larger value of E, is it not possible that E will limit the deflection so that the failures will occur in some other manner?

F. P. SHEARWOOD,[17] M. ASCE.—The practical structural designer should find this paper of unusual interest and benefit as it calls attention to that very

NOTE.—This paper by Marshall Holt and J. W. Clark was published in December, 1949, *Proceedings*. The numbering of footnotes, equations, and tables in this Separate is a continuation of the consecutive numbering used in the original paper.

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important part of structural framing—namely, the connections of compression members. Connections are often proportioned only to equal the area of the member with disregard to other factors, especially stiffness and stability. In common practice, planes of reduced section are studiously avoided, but reduced stiffness or interference with the even flow of stress is often overlooked. These are of almost equal importance, especially with compression members. These tests are excellent examples to illustrate the weakness of compression members which have planes of reduced stiffness in their connections.

TABLE 8.—SECTION MODULI OF STRUTS AND CONNECTIONS

Dimension	Section modulus[a]
(*a*) STRUTS	
One—3½ in. by 3½ in. by ¼ in.	0.79
One—3½ in. by 3½ in. by ¼ in.	0.40
Two—2½ in. by 2 in. by ¼ in.	0.58
One—2½-in. by 0.138-in. tube	0.45
(*b*) CONNECTIONS	
4½ in. by ½ in. flat	0.047
4½ in. by ½ in. flat	0.187
3½ in. by ½ in. flat	0.146
6½ in. by ½ in. flat	0.068
6½ in. by ½ in. flat	0.27
6½ in. crimped flat	0.3[b]

[a] Section moduli are for the entire main strut in each case; for example, in the third item of Table 8(*a*) 0.58 is the section modulus of two, 2½-in. by 2-in. by ¼-in. angles. [b] Approximately.

The paper also emphasizes that columns are seldom either pin ended or fixed; they vary from that having a point of contraflexure at one or both ends to that having completely rigid end supports.

So-called pin joints—those with no resistance to turning— do not occur except in the testing laboratory, because even a pin joint has turning resistance due to the friction of the pin against its bearing.

Fixed ends, or rather the fixedness of ends, are dependent on the bending resistance of the weakest section in their connections and the flexural length of the strut by the distance between these weak sections.

The section moduli of the struts given in this paper and the weak sections in their connections are approximately as shown in Table 8. These section moduli show that the bending resistance of the connections is much weaker than the main section; therefore it must influence the rigidity of the strut and consequently its compression value.

All the tests made with knife-edge bearings—namely, specimens A1 to H1, have at least one other plane of weakness to turning. If these planes had no turning resistance, the connections would be unstable; but all of them have some turning resistance. Therefore, the resistance of these struts is influenced by, and probably limited to, the turning resistance of the plane of weakness of its connections.

The erratic results of these tests are rather confusing; for instance, in comparing the F-tests with the others, why should specimen F1 with knife-edge bearings be so weak, whereas the same make-up with flat bearings gives nearly the highest results? Perhaps there is the possibility of enough difference in the setting of the struts against the bearings to account for these results.

Considering the fixed-ended struts and assuming that the rigidity of the end fixture is fully the equal of the main struts, specimens E4, E5, and H3 have no weak section and therefore should develop the full resistance as fixed end struts. Struts C2, C3, D2, D3, E2, E3, and H2 have one section which has a smaller section modulus than the main member; therefore, the point of contraflexure will

be at this weak point and its turning resistance may be the limit of the flexural resistance of the strut, and the distance between these two restricted turning points will be the length on which the strut can deflect.

Struts B2, B3, F2, F3, G2, and G3 have two weak sections, and if these sections had no turning resistance, the struts would be unstable; but both of these weak sections have some turning resistance to overcome before the strut fails, and therefore they provide some strength to the struts.

It is very difficult to visualize clearly or to estimate the resulting effect of the bending resistance of connections which have two or more planes that are weak in turning. For instance, it is hard to follow the reasons for the comparatively high test value of F2 and F3 which have very weak planes where the angle is flattened to $\frac{1}{4}$ in. This may be because all the bending moments from eccentricities happen to balance one another precisely and no flexural stresses are set up; in this particular case the strut has only direct stress to resist as long as an exact balance is not upset.

Specimens G2 and G3 have weak sections at relatively the same location, but in this case the strut angles are not completely flattened so that the section has considerably greater bending resistance.

In comparing specimens B, C, D, and H, which have only $\frac{1}{4}$-in. plates, with specimens A, E, F, and G, which have $\frac{1}{2}$-in. plates, it must be realized that the connection plates of the former are enclosed by the struts whereas the plates of the latter are free to bend between the rivet heads.

The different findings between these tests and those of J. A. Van den Broek,[14] M. ASCE, in regard to the value of the flattened angles and those connected by one leg, may possibly be due to the lower modulus of aluminum as compared with steel, which would exaggerate the flexural deflection as a result of eccentricity.

Tubular sections being symmetrical on all axes are fundamentally preferable to unsymmetrical shapes, such as angles, channels, and even I-beams, but the past and existing construction methods demand connections which do not easily suit tubes. With modern welding and cheaper tubular sections, the ideal compression section may soon come into greater use.

To compute the actual value of such connections, even approximately, as shown in the paper (even if all the unknown inaccuracies of fabrication, assembling, etc., were eliminated) is practically impossible, but this paper and the tests have emphasized, most usefully, the advisability of carrying the full section of compression members right into their reaction points, or with as little interference as possible with the straight path of their stresses.

S. K. GHASWALA,[15] Assoc. M. ASCE.—The paper forms an interesting account of the study of end connections of aluminum struts. In theory, end restraint has a very simple effect—namely, to change the effective column length. In practice, however, the accurate determination of the effects of end connections for any column except a pin-ended column is a difficult problem, especially in lattice structures like towers, booms, and light trusses. The

[14] "Effect of Connections and Rivet Holes on Ductility and Strength of Steel Angles," by J. A. Van den Broek, *Civil Engineering*, February, 1940, p. 83.

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author states that in such structures, members are connected by bolts or rivets through one leg which produces a loading eccentricity. The writer fully agrees with this fact and ventures to condemn this method as bad connection detail. This attachment produces high eccentricity, excessive stress concentration near the rivet hole and close to the edge of the angle leg, and a concentration of ductile flow near the hole. The material therefore does not develop its full strength, but ruptures through the hole before the elastic-limit stress in the main part of the member is reached. This is probably one of the main reasons why most of the test values of column strength found by the author fall well below the theoretical values computed by Eqs. 1, 2, and 3.

In evaluating the column strength, P/A, for 52S-H32 tubing (based on the compressive yield strength of 24,200 lb per sq in.), the author has used Eq. 1. This alloy[19] has a compressive yield strength of 27,000 lb per sq in. and, when $\frac{K\,L}{r}$ is less than 100, the column strength is given by

$$\frac{P}{A} = 30{,}600 - 204\,\frac{K\,L}{r} \qquad (4a)$$

For values of $\frac{K\,L}{r}$ greater than 100,

$$\frac{P}{A} = \frac{102{,}000{,}000}{\left(\frac{K\,L}{r}\right)^2} \qquad (4b)$$

In the case of 17S-T4 alloy, this verification is not possible as no specifications are given for this type. However Eq. 3 for 17S-T4 alloy for 2.5-in. by 2-in. by $\frac{3}{16}$-in. angles is the same for 61S-T alloy.[20] These slightly varying expressions require some elucidation to avoid confusion.

The study of design methods for aluminum alloy compression members, with particular emphasis on light triangulated structures, was suggested during 1949 by C. Marsh.[21] The formula of J. Perry widely used in England for steel design is modified to cover all initial imperfections and fixity conditions by an initial curvature, whereas latticed struts are analyzed by providing for distortion due to shear. Comprehensive investigations have also been conducted on the foregoing lines on the strength of aluminum struts by J. F. Baker, Assoc. M. ASCE, and J. W. Roderick of Cambridge University,[22] in Cambridge, England. In evaluating end restraint effects, aircraft structural analysis is worth studying as practically the entire gamut of restraining variables are met within this field. The situation arising here is well surveyed by Paul E. Sandroff,[23] and the subject of column end fixity coefficients has been studied by many.[24] This latter concept has very clearly brought to light (as shown by Theodor von Kármán,[25] Hon. M. ASCE) the facts that efficiently

[19] "Alcoa Structural Handbook," Aluminum Co. of America, Pittsburgh, Pa., 1948, p. 37.

[20] "Alcoa Structural Handbook," Aluminum Co. of America, Pittsburgh, Pa., 1945, p. 37.

[21] *Journal*, Soc. of Engrs., Inc., London, April, 1949, p. 15.

[22] "The Strength of Light Alloy Struts," by J. F. Baker and J. W. Roderick, *Research Report No. 3*, Aluminum Development Assn., London, July, 1948, p. 148.

[23] "Notes on Columns," by Paul E. Sandroff, *Journal of the Aeronautical Sciences*, January, 1944, p. 1.

[24] "Notes on Column End-Fixity Coefficients," by G. C. Best, *ibid.*, February, 1949, p. 117.

[25] *Zeitschrift für Architectur und Ingenieurwesen*, 1889.

designed structures cannot be made proportionately stronger by just increasing the amount of end restraint and, secondly, that end restraint causes a marked increase in buckling strength only for long slender columns and not for short ones. F. R. Shanley[26] has conducted exhaustive studies and the results of his investigations for 24-ST aluminum alloy columns with varying fixity coefficients is worth noting. An excess of actual fixity over the assumed value can do little good, but lack of fixity (with the same assumed value) can do much harm. End restraint in an eccentric column tends to reduce the effective eccentricity. In the case of columns loaded within the elastic limit this reduction increases with loading and assumes pronounced results in the inelastic range. Small eccentricities, therefore, do not materially affect the structural strength. The combined effects of large eccentricities and end restraint are a subject for further research and experimentation as still many aspects remain unsolved. The authors' investigations on the comparative strength of various sections appear to bridge these gaps to some extent, for light alloy structures. Actual tests can only reveal the validity of the assumption that these findings are also applicable to members of other materials.

MARSHALL HOLT,[27] ASSOC. M. ASCE, AND J. W. CLARK,[28] JUN. ASCE.—In helping to clarify the significance of the test results, the discussers have added materially to the paper. Mr. Ghaswala's footnote references are noteworthy.

The analysis by Mr. Shearwood of the variations in stiffness of the struts and their connections is a concise picture of the weaknesses that contributed to the failure of the various members. He raises the question of why specimen F1, with ends completely flattened, was the weakest of the single-angle specimens tested on knife-edge bearings, whereas specimen F2 was one of the strongest of the members loaded as flat-end columns. The comparatively high strength of specimen F2 with flat ends can probably be explained by the fact that the stress measurements indicated less effective eccentricity for the type F connection than for the other single-angle connections. The strength of the round end columns, which failed by the formation of double hinges at the ends, was governed to a large extent by the stiffness of the weak sections that acted as hinges. As stated by Mr. Shearwood, the flattened end of specimen F1 formed a more effective double hinge than the connections of the other round end, single-angle columns. This weakness resulted in a lower ultimate strength.

Some of the discussers question the writers' belief that the general conclusions concerning the relative merits of various types of end connections may be applicable to materials other than aluminum. Although it is true that the strength of a member depends on the material of which it is made, the relative stiffness of various types of columns and connections and the relative degrees of eccentricity and of stress concentration are functions of geometry only. For this reason it seems logical that a comparative evaluation of these factors based on tests of aluminum alloy columns may also be valid for columns of other materials.

[26] "Applied Column Theory," by F. R. Shanley, *Transactions*, ASCE, Vol. 115, 1940, p. 698.

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Mr. Harland discusses the differences in some of the mechanical properties of steel and aluminum. In this connection it is important that no ill effects resulting from the relatively lower ductility (as measured in the tensile test) of the aluminum alloys as compared with steel were manifest in the tests reported in the paper. Except in the case of specimen E3 where the rivets sheared, there was no parting of material in any of the static tests. The alloys used in these tests were capable of undergoing the elongations consistent with the lateral deflections of the members. As noted in the case of specimens E1 and G1, the loading was stopped because the deformations were so great that there was danger that the specimen would fly out of the testing machine. It is true that the elastic lateral deflections of steel specimens of the same dimensions would be smaller than those encountered in these tests, but the ultimate failures of such steel members would undoubtedly also be accompanied by sidewise bending. The average stresses would probably be above the elastic stress range, as was the case with specimens A3 and C3. The authors know of no series of tests that demonstrates the amount of ductility actually required of the material in structural parts of this type.

Mr. Ghaswala suggests that the poor agreement between the test data and the computed curves of Eqs. 1, 2, and 3 is a result of material failure produced by concentration of stress around rivets. This certainly is not the case, but rather the poor agreement results from the eccentricities of loading that are not considered by these equations. Reconciling Eqs. 1 and 4 is quite simple when it is realized that Eq. 1 is based on the yield strength of a certain piece of material and Eq. 4 is based on a typical value obtained for that alloy from production records over a period of time from a variety of shapes produced by a variety of manufacturing processes.

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